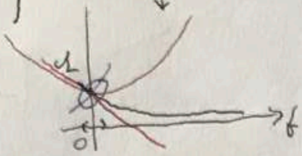
EX. $v(t) = \frac{mg}{h}\left(1 - e^{-\frac{h}{m}t}\right), \quad t \geq 0$

ANALISI A 23/10/20

1

$(m, g, h > 0)$



SVILUPPI DI TAYLOR CON RESTO DI PEANO

PRIMI ISTANTI DI CADUTA $t \approx 0$

$|x| \leq \varepsilon$

ε "piccolo"

$\underset{x \approx 0}{e^x} = 1 + x + o(x)$ (!) $= 1 + x + \frac{x^2}{2} + o(x^2)$ $(x \to 0)$

$-ky, \ k > 0 \quad \frac{d}{dy}(e^{-ky}) = -k e^{-ky}$

$e^{-\frac{h}{m}t}$ come se $x = -\frac{h}{m}t$

$o(t)$

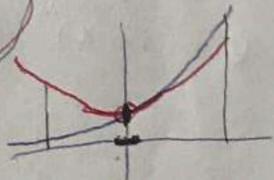
$\underset{t \approx 0}{\exp\left(-\frac{h}{m}t\right)} = 1 - \frac{h}{m}t + \frac{1}{2}\frac{h^2}{m^2}t^2 + o(t^2)$

$t > 0$

$\Rightarrow \quad v(t) = \frac{mg}{h}\left(\frac{h}{m}t + o(t)\right)$

$= gt + o(t)$

PER ECCESSO

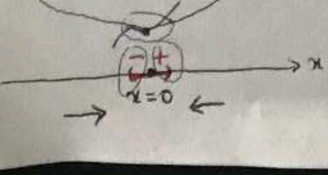


$\Rightarrow$

$v(t) = \frac{mg}{h}\left[\frac{h}{m}t - \frac{1}{2}\frac{h^2}{m^2}t^2 + o(t^2)\right]$

$= gt - \frac{gh}{2m}t^2 + o(t^2)$. $\boxtimes$

$t > 0$



(2

$(1+x)^n \geq 1 + nx$ $\quad x \geq -1$ (FISSATO)

$\boxed{\text{EX.}}$

$n \geq 1$

DIM. PER INDUZ. (SU n).

$\boxed{-1 \leq x < 0}$:

$(1+x)^n \geq 0 \overset{?}{\geq} 1 + nx$ NON $\forall n$

$n=0$: $\underbrace{(1+x)^0}_{1} \geq \underbrace{1+0\cdot x}_{1}$?

$n=1$: $1+x \geq 1+x$ (= !)

? $\underbrace{(1+x)^n \geq 1 + nx}$ $\overset{???}{\Rightarrow}$ $(1+x)^{n+1} \geq 1+(n+1)x$?

$(1+x)^{n+1} \equiv (1+x)^n(\underbrace{1+x})$

$= \underbrace{(1+x)^n} + x\cdot(1+x)^n$

$a \geq b$
$\downarrow$
$a+c \geq b+c$

($a\cdot(b+c) = a\cdot b + a\cdot c$
DOVE :
$a \doteq (1+x)^n$, $b \doteq 1$, $c \doteq x$)

$\geq \underbrace{(1+nx)} + \underbrace{x(1+x)^n}$

IPOTESI INDUTTIVA

$x(1+x)^n \geq x$ SOLO SE $x \geq 0$ —

SE $x \geq 0$:

$\geq (1+nx) + x = 1 + (n+1)x$

$\boxed{(1+x)^n \overset{(*)}{\geq} 1+nx}$, $n \geq 0$, $x \geq -1$ FISSATO

INDUZIONE (SU n); $x \geq 0$ OK

PER $-1 \leq x < 0$? (*) È VERA PER n ABBASTANZA GRANDE. ~~NON PUÒ ESSERE VERA PER OGNI n~~

INFATTI, $\forall\, x \in [-1, 0[$ FISSATO,

$$1+nx \downarrow -\infty \quad \text{PER } n \to \infty$$

$\boxed{\text{ES}}$ $x = -\frac{1}{2}$: $1+nx = 1 - \frac{n}{2}$

$\Rightarrow$ PER n ABB. GRANDE, T.C. $1+nx \leq 0$,
$(n \geq n_x)$

$$(1+x)^n \geq 0 \geq 1+nx. \quad ✓ \qquad \boxtimes$$

$\boxed{\text{EX.}}$ $$\sum_{\substack{n=1,\\ n \text{ PARI}}}^{\infty} \frac{\cos(\frac{\pi}{2} n)}{n} = \sum_{m=1}^{\infty} \frac{\cos(\frac{\pi}{2} 2m)}{2m}$$

$$= \frac{1}{2} \sum_{m=1}^{\infty} \frac{\cos(\pi m)}{m}$$

$\sum_{m=1}^{\infty} a_m$, $a_m \in \mathbb{R}$, $a_m = \frac{\cos(\pi m)}{m} = \frac{(-1)^m}{m}$

$$\sum_{m=1}^{\infty} \frac{(-1)^m}{m} \qquad (-1)^m \cdot b_m, \quad b_m \downarrow 0 \quad (b_{m+1} \leq b_m)$$

$$b_m = \frac{1}{m}$$

$$|a_m| = \frac{1}{m}$$

(LEIBNIZ)

NON ASS. CONV.! $\left(\sum_{m \geq 1} \frac{1}{m} = \infty \right)$

4

EX. $\sum_{n=2}^{\infty} \frac{\sin(\log n)}{n^2 \log n} = a_2 + \sum_{n=3}^{\infty} a_n \leq a_2 + \sum_{n=3}^{\infty} \frac{1}{n^2} < +\infty$

$a_n \doteq \frac{\sin(\log n)}{n^2 \log n} \in \mathbb{R}$, $|a_n| \leq \frac{1}{n^2 \log n} \leq \frac{1}{n^2}$

$n^2 \leq n^2 \log n$, $\log n \geq 1$, $n \geq e$ $(n \geq 3)$

CI BASTAVA $\forall n$ ABB. GRANDE (OK) $|a_n| \leq \boxed{\frac{1}{n^2 \log n} \leq \frac{K}{n^2}}$, $K > 0$, IN MODO CHE

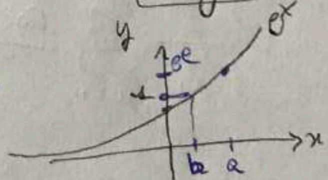
CRITERIO CONFRONTO $\to$ $\sum_{n=2}^{\infty} |a_n| \leq K \sum_{n=2}^{\infty} \frac{1}{n^2} < +\infty$

PROVIAMO $K = 1$:

PER QUALI n $\frac{1}{n^2 \log n} \leq \frac{1}{n^2}$?

CIOÈ, $n^2 \log n \geq n^2$? $\boxed{\log n \geq 1}$?

$a \geq b \iff e^a \geq e^b$



STRETT. CRESC.

$e^{\log n} \geq e^1 \iff n \geq e = 2.7\ldots$

$\Updownarrow$

$\boxed{n \geq 3}$

" $\sum_{n=n_0}^{\infty} a_n$ " $= \lim_{n \to \infty} S_n$, $S_n \doteq \sum_{m=n_0}^{n} a_m$

$\to$ S_n PER n ABB. GRANDE

$\sum_{n=0}^{\infty} (-1)^n$ $\nearrow \underbrace{(1+(-1))}_{0} + \underbrace{(1+(-1))}_{0} + (\ldots)$ 0

$\searrow 1 + \underbrace{(-1+1)}_{0} + \underbrace{(-1+\ldots)}_{0}$ 1

?

EX

$$\lim_{x\to 0} \frac{x\sin x + \log(1-x^2)}{x^2[2x+x^2]^2}$$

$+\frac{x^5}{5!}$

$\mathcal{O}(x^3)$ "$\mathcal{O}(x^4) = \mathcal{O}(x^3)$"

PER $x \simeq 0$ $(x \neq 0)$, $\sin x = x - \frac{x^3}{6} + \mathcal{O}(x^4)$ $\qquad \frac{\mathcal{O}(x^4)}{x^4} \cdot x$ PER $x \to 0$ $\quad \downarrow 0 \quad \downarrow 0$

PER $z \simeq 0$, $(|z| \leq \varepsilon)$ $\log(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} + \mathcal{O}(z^3)$

$$\log(1 + \underbrace{(-x^2)}_{\text{"}z\text{"}}) = -x^2 - \frac{x^4}{2} - \frac{x^6}{3} + \mathcal{O}(x^6)$$

DE L'HÔPITAL. $\left(\frac{0}{0}\right)$ $f(x) \doteq x\sin x + \log(1-x^2)$

$g(x) \doteq x^2[2x+x^2]^2$

$= x^2[x^4 + 4x^3 + 4x^2]$

$= x^6 + 4x^5 + 4x^4$

x^2

x^4

$$g'(x) = 6x^5 + 20x^4 + 16x^3 \qquad 96$$

$$f'(x) = \sin x + x\cos x + \frac{1}{1-x^2}\cdot(-2x)$$

$$= \sin x + x\cos x - \frac{2x}{1-x^2}$$

$(1-x^2)^2$

$-\cos x$

$$\lim_{x\to 0} \frac{f'(x)}{g'(x)} \quad 0/0$$

...

$x \to x_0 = 0$

$$x\sin x = x^2 - \frac{x^4}{6} + \mathcal{O}(x^5) \qquad (\mathcal{O}(x^4)\cdot x = \mathcal{O}(x^5))$$

SVILUPPI DA "RICORDARE":

e^x, $\sin x$, $\cos x$, $(1+x)^\alpha$, $\arctan x$, $\log(1+x)$

$$(1+x)^\alpha \underset{x\to 0}{=} 1+\alpha x+\binom{\alpha}{2}x^2+o(x^2) \qquad (x\geq -1)$$

$\boxed{\alpha=-1}$
$$\frac{1}{1+x} \underset{x\to 0}{=} 1-x+x^2+o(x^2)$$

$\boxed{\text{ES}}$ $\exists \lim_{x\to 0}\left[\frac{1}{x}-\frac{1}{\sin x}\right]$?

$$\frac{\sin x - x}{x\sin x}$$

TAYLOR "AL VOLO"

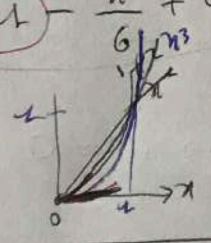
DE L'HOSPITAL 2 VOLTE ALMENO

$(x\cos x+\sin x)$

$$\sin x \underset{x\to 0}{=} x-\frac{x^3}{6}+o(x^3)$$

$$\Rightarrow \frac{\sin x - x}{x\sin x} = \frac{-\frac{x^3}{6}+o(x^3)}{x^2-\frac{x^4}{6}+o(x^4)} \qquad x^2\left(1-\frac{x^2}{6}+o(x^2)\right)$$

$$= \frac{-\frac{x}{6}+o(x)}{1-\frac{x^2}{6}+o(x^2)} \xrightarrow[x\to 0]{} \frac{0}{1}=0$$



$x^n<1$ $\left(x=\frac{1}{y^n}\right)$, $y>1$

$\boxed{1}$

$\boxed{\text{EX.}}$ [SERIE] DIM. CHE: (a) $\sum_{n=1}^{\infty}\frac{1}{n\sqrt[n]{n}}=\infty$ (b) $\sum_{n=0}^{\infty}3^{-\sqrt{n}}<\infty$

(PER CASA)

(c) $\sum_{n=1}^{\infty}\left(\frac{1}{2n-1}-\frac{1}{2n}\right)$ CONV., $\sum_{n=1}^{\infty}\left(\frac{1}{2n-1}-\frac{1}{n}\right)$ DIV., (d) $\sum_{n=3}^{\infty}\frac{(-1)^{n-1}}{\log(\log n)}<\infty$.

ANALISI A
6/11/2020

$$\lim_{n\to\infty}\left(\frac{n+2}{n-1}\right)^{2n} = e^6 \qquad \boxed{\text{EX.}}$$

$$\cancel{e^{\log(1^{+\infty})}} \qquad e^{\log 1^{+\infty}} = e^{(+\infty)\cdot(\log 1)^{0}} \quad] \; 0\cdot(+\infty)$$

I° MODO

$$= \exp\left\{\underbrace{\log\left(\frac{n+2}{n-1}\right)^{2n}}\right\}$$

$$2n\cdot\log\left(\frac{n+2}{n-1}\right) \quad \cancel{\text{NO}}\ \ldots$$

$$\lim_{n\to\infty}\sqrt[n]{n} \qquad \exp\{\log\sqrt[n]{n}\} \to 1 \qquad \log n^{1/n} = \frac{\log n}{n} \to 0$$

$$6\cdot o(1) = o(1)$$

$$\left(\frac{n+2}{n-1}\right)^{2n} \overset{*}{=} \left[\left(1+\frac{3}{n-1}\right)^{\frac{n-1}{3}}\right]^{\frac{3}{n-1}\cdot 2n}$$

$$\frac{n+2}{n-1} = \frac{(n-1)+3}{n-1} = 1+\frac{3}{n-1}$$

$$= 2n\cdot\log\left(\frac{n(1+2/n)}{n(1-1/n)}\right)$$

$$(e^b)^c = e^{b\cdot c}$$

$$\forall c\in\mathbb{R},\ c\,o(\ldots) = o(\ldots)$$

$$\underbrace{o\left(\tfrac{1}{n}\right)+o\left(\tfrac{1}{n}\right)}_{=\,2o\left(\tfrac{1}{n}\right)} = o\left(\tfrac{1}{n}\right)$$

$$\log(1+x) \underset{x\to 0}{=} x + o(x)$$

$$o(1) \qquad \frac{o(1)}{1}\to 0$$

$$= 2n\left[\underbrace{\log\left(1+\tfrac{2}{n}\right)}_{\frac{2}{n}+o\left(\frac{1}{n}\right)} - \underbrace{\log\left(1-\tfrac{1}{n}\right)}_{-\frac{1}{n}+o\left(\frac{1}{n}\right)}\right]$$

$$= 2n\left[\underbrace{\frac{2}{n}+\frac{1}{n}}_{3/n} + o\left(\frac{1}{n}\right)\right]$$

$$\underbrace{\frac{|o(\frac{1}{n})|}{\frac{1}{n}}}\underset{n\to\infty}{\longrightarrow} 0$$

$$= 6 + o(1) \quad \checkmark$$

$$o(1) = \boxed{n\,o\left(\tfrac{1}{n}\right)} \underset{n\to\infty}{\longrightarrow} 0$$

$$\left(1+\frac{3}{n-1}\right)^{\frac{n-1}{3}} \overset{*}{=} \left(1+\frac{1}{\frac{n-1}{3}}\right)^{\frac{n-1}{3}} \to e$$

II° MODO

$$\left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e\right]$$

$$\frac{3}{n-1}\cdot 2n = \frac{6n}{n-1} = \frac{6\cancel{n}}{\cancel{n}(1-1/n)} = \frac{6}{1-1/n} \xrightarrow[n\to\infty]{} 6 \; . \; \boxed{}$$

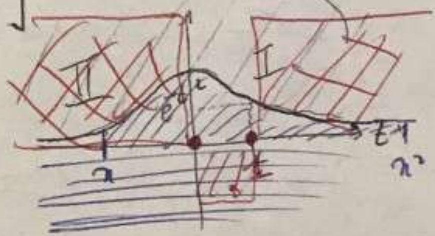
$\boxed{\text{EX:}}$

$$F(x) \doteq \int_{x}^{x^2} \underbrace{e^{-t^2}}\,dt \quad , \; x \in \mathbb{R} \; . \; \underline{\text{Per iniziare}}, \; F(0)=0 \;\&\; F(1)=0 !$$

$\forall t, \; \exp(-t^2) > 0 \to F(\cdot) \geq 0$, anzi $F(x) > 0 \; \forall x \neq 0$, QUANDO $x < x^2$.

[$\forall I \subset \mathbb{R}$ INTERVALLO ("ORIENTATO POSITIVAMENTE") $\neq \emptyset$, DATA $f: I \to \mathbb{R}$, SE $f(\cdot) \geq 0$ E SE

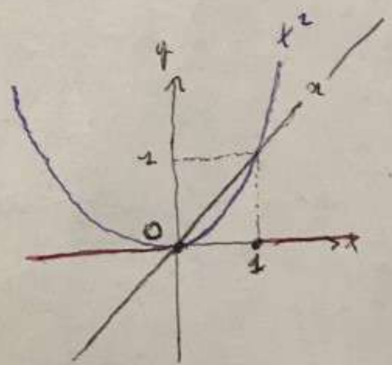
$\int_I f(t)\,dt = 0$, ALLORA $f \equiv 0$.]



$f(t) := e^{-t^2} \in C^\infty \to F \in C^\infty$.

NON È UN PROBLEMA SE $x<0$!

$\int$ umme (dt), $\sum_{n=1}^{N}$, $\sum_{n=1}^{\infty}$
" "
$t \in [x, x^2]$



$\boxed{0 < x \leq 1}$

$x<0 \to x < x^2$
$x>1 \to x < x^2$

$\to \boxed{x^2 < x}$

$$\to F(x) \overset{\text{def.}}{=} \int_x^{x^2} e^{-t^2}\,dt = -\int_{x^2}^{x} e^{-t^2}\,dt \quad \text{per } 0 \leq x \leq 1$$

SCRIVERE NELLA FORMA $\int_a^b$, $a<b$

(ABBIAMO USATO ANCHE:

$$\int_a^a (\dots)\,dt = 0 \quad , \; \forall a \in \mathbb{R} .)$$

(3

$$F(x) = \begin{cases} \int_x^{x^2} e^{-t^2}\,dt, & \text{se } x \le 0 \\ \boxed{-\int_{x^2}^{x} e^{-t^2}\,dt,} & \text{se } 0 \le x \le 1 \\ \int_x^{x^2} e^{-t^2}\,dt, & \text{se } x \ge 1 \end{cases} \qquad F(0)=F(1)=0$$

$F|_I$, $I \subset \mathbb{R}$ APERTO (TIPO $]-\infty,0[$, $]0,1[$, ---)

"POSSIAMO DERIVARE"

$\lim_{x \to x_0}$ 0 1 x_0 ...

solo da sinistra, in questo caso

...

[PROVATE A CONTINUARE VOI]

EX.

$$I := \int \frac{x+2}{2x-3}\,dx = \frac{x}{2} + \left(\int \frac{2}{2x-3}\,dx\right)\left(1+\frac{3}{4}\right) \quad \frac{7}{4} \quad + c$$

$\log(2x-3)$

$$\frac{1}{2} + \frac{2+3/2}{2x-3}$$

$$\frac{(2x-3)+4}{2\cdot(2x-3)} \to 2x+1 \quad \frac{2x+1}{2(2x-3)}$$

$\boxed{t := 2x-3}$, $x = \frac{t}{2} + \frac{3}{2}$, $dx = \frac{1}{2}dt$ [e stesso "ORIENTAZIONE"]

$$I = \int \frac{1}{2t}\left\{\frac{t}{2} + \frac{7}{2}\right\} dt$$

$$= \int \left[\frac{1}{4} + \frac{7}{4t}\right] dt = \frac{t}{4} + \frac{7}{4}\log|t|$$

$$\frac{x}{2} + \frac{7}{4}\log|2x-3| + c$$

$\boxed{\text{EX.}}$

$$\int \frac{x-1}{x^2+2x+5}\,dx = \frac{1}{2}\int \frac{2x-2}{x^2+2x+5}$$

$$2x-2 = (2x+2)-4$$

DENOMINATORE IRRIDUCIBILE!

$$= \frac{1}{2}\left\{ \underbrace{\int \frac{2x+2}{x^2+2x+5}\,dx} - \int \frac{4}{x^2+2x+5}\,dx \right\}$$

$$\int \frac{4}{x^2+2x+5}\,dx = \int \frac{1}{\left(\frac{x+1}{2}\right)^2 + 1}\,dx \qquad \frac{x+1}{2} = t$$

$$x^2+2x+5 = (x+1)^2 + 4$$

$$\left[\int \frac{dt}{1+t^2} = \arctan(t)\right]$$

$$= 2\underbrace{\int \frac{1/2}{\left(\frac{x+1}{2}\right)^2+1}\,dx}_{\arctan\left(\frac{x+1}{2}\right)}$$

EX. (PER CASA) STUDIO QUALITATIVO: $f(x) \doteq \dfrac{|e^x-1|}{1+|x|}$, $x \in \mathbb{R}$.

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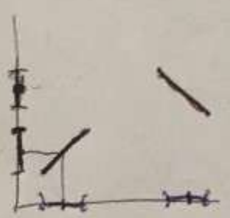
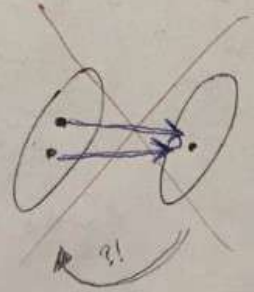
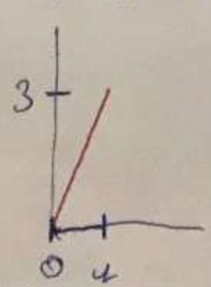
mtersio1@uninsubria.it ← MIA E-MAIL!







COMMENTI BIUNIVOCITÀ (INIETTIVITÀ) E STRETTA MONOTONIA (di funzioni continue/derivabili)......

$\boxed{\text{EX}}$ ["RITOCCATO FUORI ORARIO DI LEZIONE"] Dimostrare "per sostituzione" che (5

$$F(x) := \int \frac{1}{1+x^2}\,dx = \arctan x$$

(come funzioni $\mathbb{R}_x \to \mathbb{R}$) nel senso che, per ogni $s \in]-\frac{\pi}{2}, \frac{\pi}{2}[$,

$$F(\tan s) = s .$$

[Qui viene usato che $\tan(\cdot):]-\pi/2, \pi/2[_s \to \mathbb{R}$ e $\arctan(\cdot): \mathbb{R}_x \to]-\pi/2, \pi/2[$ sono (biunivoche e) una l'inversa dell'altra — e che le inverse sono UNICHE!]

SVOLGIMENTO. La tesi equivale a dimostrare che, $\forall\, t \in]-\pi, \pi[$, $F(\tan \frac{t}{2}) = \frac{t}{2}$ (prendiamo $s = \frac{t}{2}$). La sostituzione suggerita è $x = \tan \frac{t}{2}$ (regolarissima): si ha quindi $dx = \frac{1}{2\cos^2 \frac{t}{2}}\,dt$ (ed è una trasformazione che MANTIENE L'ORIENTAMENTO), dove abbiamo usato che $\frac{d}{dy}\tan(y) = \frac{1}{\cos^2 y}$, da cui

$$F(\tan \tfrac{t}{2}) \equiv \int \frac{1}{1+(\tan\frac{t}{2})^2}\,\frac{1}{2\cos^2\frac{t}{2}}\,dt$$

$$= \frac{1}{2}\int dt$$

$$= \frac{t}{2} . \quad \boxtimes$$

$\left(\leftarrow 1 + (\tan \frac{t}{2})^2 = 1 + \frac{\sin^2(t/2)}{\cos^2(t/2)} = \frac{1}{\cos^2(t/2)} \right)$

$[\cos^2\alpha + \sin^2\alpha = 1]$

ANALISI A. (13/11/20) $(\alpha>0?)$ → DUBBI SULLA RISOLUZIONE DEL PROF. .... (1)

EX. [DUBBI] $\sum_{n=1}^{\infty} \frac{n^\alpha[1-\cos(n^{1-\alpha})]}{\sin(\frac{1}{n})}$, $\alpha \in \mathbb{R}$. ? CONVERGE $\Leftrightarrow |\alpha|>2$? ERRATO

SVOLG. $\sin(\frac{1}{n})>0 \ \forall n \geq 1$, $\cos(\dots) \in [-1,1] \Rightarrow 1-\cos(n^{1-\alpha}) \geq 0$ $(\in [0,2])$

→ SERIE È A TERMINI NON-NEGATIVI (≥ 0), SIANO "a_n".

$\boxed{\alpha<-2}$: LA SERIE CONVERGE: per $n\to\infty$, $\sin(\frac{1}{n}) = \frac{1}{n} + o(\frac{1}{n}) = \frac{1}{n}[1+o(1)]$

$$\Rightarrow a_n \sim \frac{n^\alpha[1-\cos(n^{1-\alpha})]}{1/n} = \frac{1-\cos(n^{1-\alpha})}{n^{-(\alpha+1)}}, \text{ e ora}$$

$$\frac{1-\cos(n^{1-\alpha})}{n^{-(\alpha+1)}} \leq \frac{2}{n^{-(\alpha+1)}}. \text{ Ma } \alpha+1<-1, \text{ cioè}$$

$$-(\alpha+1)>1 \text{ e così } \sum \frac{2}{n^{-(\alpha+1)}}<\infty \Rightarrow \sum a_n<\infty. \checkmark$$

$\boxed{\alpha>0}$ "~~TORNA BENE~~" POSSIAMO USARE TAYLOR/MAC LAURIN ANCHE PER $\cos(n^{1-\alpha})$

SOLO PER $\boxed{1-\alpha<0}$, OSSIA $\boxed{\alpha>1}$, cosicché

$$n^{1-\alpha} = \frac{1}{n^{\alpha-1}} \text{ con } \alpha-1>0 \ (e \xrightarrow[n\to\infty]{} 0). \text{ Allora:}$$

$$\cos(n^{1-\alpha}) \underset{n\to\infty}{=} 1 - n^{2(1-\alpha)}/2 + o(n^{2(1-\alpha)})$$

$$\rightarrow 1-\cos(n^{1-\alpha}) = \frac{n^{2(1-\alpha)}}{2}[1+o(1)]$$

→ $c\cdot x^2 + o(x^2) = cx^2[1+o(1)]$

$cx^2 o(1) = o(x^2)$

"COME SE C NON CI FOSSE"

$$\rightarrow a_n \sim \frac{n^\alpha \cdot \frac{n^{2(1-\alpha)}}{2}}{n^{-1}} = \frac{1}{2} n^{2(1-\alpha)+\alpha+1}$$

$2\frac{o(x^2)}{x^2} = \frac{(2\,o(x^2))}{x^2} \xrightarrow[x\to 0]{} 0.2=0$

$2o(x^2)=o(x^2)$

$$= \frac{1}{2} n^{3-\alpha}. \text{ Allora}$$

$$\sum a_n<\infty \overset{\alpha>1}{\Leftrightarrow} \sum \frac{1}{n^{\alpha-3}}<\infty, \text{ cioè } \alpha-3>1, \ \alpha>4. \checkmark$$

QUINDI: LA SERIE CONVERGE PER $\alpha > 4$, MENTRE NON CONV. PER $1 < \alpha \le 4$, RESTA $0 < \alpha \le 1$. $[-2 \le \alpha \le 1 \dots]$

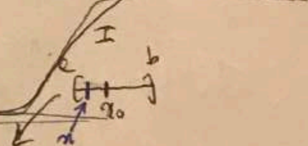
$\boxed{\alpha = 1}$ $a_n \sim \dfrac{n[1-\cos(1)]}{1/n} = n^2[1-\cos(1)] \uparrow \infty$

$\boxed{0 < \alpha < 1}$ $a_n \sim \dfrac{n^\alpha[1-\cos(n^{1-\alpha})]}{1/n} = n^{\alpha+1}[1-\cos(n^{1-\alpha})]$; $1-\alpha > 0$; $\alpha+1 > 1$

$n^{1-\alpha} \ne \frac{k}{2}\pi$ "sempre o quasi"

$n^{l/m} = \frac{k}{2}\pi \qquad \pi = \frac{2}{k} n^{l/m} \ \dots \quad \dots \dots$

$\boxed{\text{EX.}}$ $\displaystyle\int \frac{dx}{1+x^2} = \operatorname{arctan} x$ (SOSTITUZIONE).



$\boxed{\text{EX.}}$ $F(x) := \displaystyle\int_x^{x^2} e^{-t^2}\,dt$, $x \in \mathbb{R}$.

$= \displaystyle\int_0^{x^2}(\dots) - \int_0^{x}(\dots)$

$\begin{cases} F(x) = \displaystyle\int_{x_0}^{x} f(t)\,dt \\ f: I \to \mathbb{R} \text{ cont.}, I \text{ INTERVALLO}, x_0 \text{ FISSATO} \end{cases}$

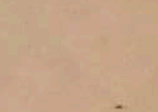
$\Downarrow$

$F'(x) = f(x) \ \forall x \in I$

(NON IMPORTA CHI SIA x_0!)

$\forall k \in \mathbb{R}, (k>0)$

$\boxed{\text{EX.}}$ Dim. $\displaystyle\int_{-k}^{k} f(x)\,dx = 2\int_0^k f(x)\,dx$, SE f PARI (e continua) SU $\mathbb{R}$



SVOLG. $\displaystyle\int_{-k}^{k} f(x)\,dx = \int_{-k}^{0} f(x)\,dx + \int_0^k f(x)\,dx$; ora verifichiamo che

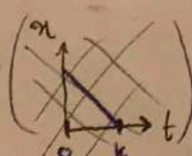
$$\int_{-k}^{0} f(x)\,dx = \int_0^k f(x)\,dx.$$

$\forall k > 0$ ~~$\forall k \in \mathbb{R}$~~, $\displaystyle\int_{-k}^{0} f(x)\,dx = -\int_0^k f(x)\,dx$.)

$x := -t$

$\| $

$\varphi(t)$



$dx = -dt$, ora ~~$\forall k$~~ $x = 0 \leftrightarrow t = 0$, $x = -k \leftrightarrow t = k$; $\varphi^{-1}(0) = 0$, $\varphi^{-1}(-k) = k$

$\frac{d}{dt}\varphi(t) = -1$

$$\Rightarrow \quad \int_{-k}^{0} f(x)\,dx = \int_{k}^{0} \underbrace{f(-t)}_{f(t)}(-dt) = -\int_{k}^{0} f(t)\,dt$$

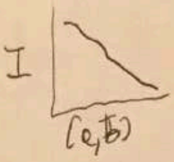
$$= \int_{0}^{k} f(t)\,dt. \checkmark \qquad \varphi(t) = t$$

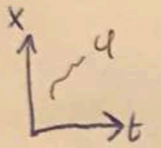
$$\int_0^k f(t)\,dt = \int_0^k f(x)\,dx$$

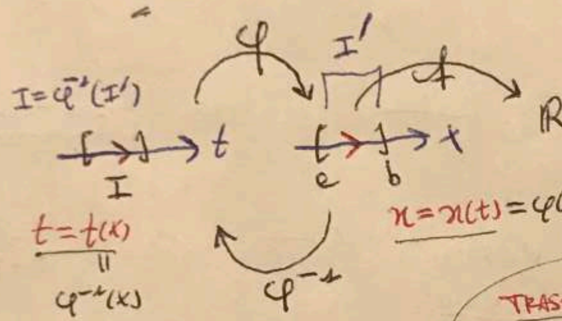
~~Schema~~ SOSTITUZIONE:

$$\begin{cases} f: [a,b]_x \to \mathbb{R} \text{ CONTINUA } (a<b) \\ \varphi: I_t \to [a,b]_x \text{ "DIFFEOMORFISMO } C^1\text{" } [\varphi \in C^1, \exists \varphi^{-1}, \varphi^{-1} \in C^1] \end{cases}$$

$$\Rightarrow \quad \int_a^b f(\underset{=}{x})\,dx = \int_{\varphi^{-1}(a)}^{\varphi^{-1}(b)} f(\varphi(t))\,\varphi'(t)\,dt.$$







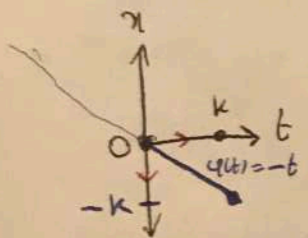
$$\varphi'(t) = x'(t) = \frac{dx}{dt}(t)$$

TRASFORMAZIONE LUNGHEZZE INFINITESIME: $dx = |\varphi'(t)|\,dt$

$$\Downarrow$$

$$\int_{I'} f(x)\,dx = \int_I f(\varphi(t))\,|\varphi'(t)|\,dt$$

$$dx = \left|\frac{dx}{dt}(t)\right| dt$$



$\boxed{\text{EX.}}$ $\int e^x (\sin x)^2\,dx$: PER PARTI!

$$\left(\frac{1}{2}(1 - \cos 2x) \rightarrow \int e^x \cos(2x)\,dx \quad \text{ABBASSARE IL GRADO} \right)$$

$\boxed{\text{EX.}}$ (PER CASA)

① $(\forall n \in \mathbb{N}^*,)$ $\forall a_1, \dots, a_n \in \mathbb{R}$, $(a_1 + \dots + a_n)^2 \le n(a_1^2 + \dots + a_n^2)$. [STIMA SENZA I DOPPI PRODOTTI!]

② $\forall f \in C^1(\mathbb{R})$, $\forall x, \bar{x} \in \mathbb{R}$, $f(x) = f(\bar{x}) + (x - \bar{x}) \int_0^1 f'(\bar{x} + \theta(x - \bar{x}))\,d\theta$.

③ $g \in C^2(\mathbb{R})$, $x, \bar{x} \in \mathbb{R}$,

$$g(x) = g(\bar{x}) + g'(\bar{x})(x - \bar{x}) + (x - \bar{x})^2 \int_0^1 \theta\, g''(x + \theta(\bar{x} - x))\,d\theta$$

[~~DA FARE SENZA FRETTA...~~]

$\boxed{\text{EX.}}$ (PER CASA)

① $\forall y_0 > 0$, $\displaystyle\int_{y_0}^{\infty} \frac{1}{x^2 \log(1+x^2)}\,dx < \infty$.

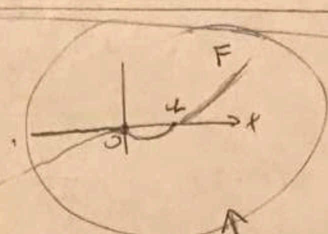
$(\varepsilon > 0)$ $\Rightarrow \displaystyle\int_{\varepsilon}^{\infty} (\dots)\,dx$

ESCLUDIAMO LO ZERO

② $\forall y_0 > 0$, $\displaystyle\int_{y_0}^{\infty} \frac{1}{x \log(1+x^2)} = \infty$.

$\boxed{\text{EX.}}$ [DUBBI DI SIMMETRIA]

$$F(x) \doteq \int_{x^2}^{x^3} \log^2(1+t^2)\,dt \qquad (x \in \mathbb{R}).$$



NON SARÀ SIMMETRICA.....

$$= \underbrace{\int_{x^2}^{0} (\dots)\,dt}_{\text{PARI OK}} + \int_{0}^{x^3} (\dots)\,dt$$

$$\left(-\int_{e}^{b} (\dots)\,dt = \int_{b}^{e} (\dots)\,dt \right)$$

$$G(x) \doteq -\int_{0}^{x^2} (\dots)\,dt \Rightarrow G(-x) = G(x). \quad \text{(PARI)}$$

$(-x)^2 = x^2$

$\Rightarrow H(x) := \displaystyle\int_0^{x^3} \log^2(1+t^2)\,dt$. $\forall x \in \mathbb{R}$, $H(-x) = H(x)$?

cioè $\displaystyle\int_0^{x^3} \log^2(1+t^2)\,dt = \int_0^{-x^3} \log^2(1+t^2)\,dt$?

$$\left(= -\int_{-x^3}^{0} \log^2(1+t^2)\,dt \right)$$

RICORDIAMO CHE:

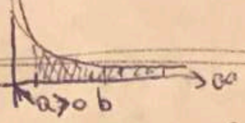
$f(t) := \log^2(1+t^2)$ PARI $\Rightarrow$

$\Rightarrow \forall k \in \mathbb{R}$, $\displaystyle\int_{-k}^{0} f(t)\,dt = \int_0^k f(t)\,dt$.

$\Rightarrow$ $\forall x \in \mathbb{R}$, ponendo $k := x^3$,

$$\underbrace{\int_0^{x^3} f(t)\,dt}_{\substack{\| \\ H(x)}} = \underbrace{\int_{-x^3}^{0} f(t)\,dt}_{\substack{\| \\ -\int_0^{-x^3} f(t)\,dt \,=:\, H(-x)}} = -H(-x)$$

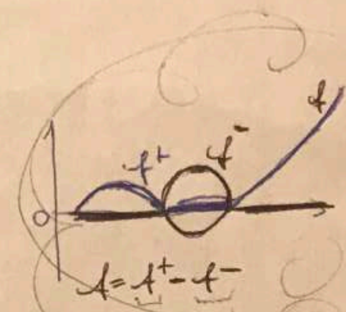
Cioè, $\forall x \in \mathbb{R}$, $H(x) = -H(-x)$ ossia $H(-x) = -H(x)$, ossia H è dispari. Dunque $F = G + H$ è né pari né dispari. $\blacksquare$



~~RIPASSO INTEGRAZIONE SU I ILLIMITATO~~

Dato $a \in \mathbb{R}$ e data $f: [a, \infty[\to \mathbb{R}$ continua, sappiamo che $\forall b \in \mathbb{R}$ con $a < b$, esiste finito $\int_a^b f(x)dx$. Definiamo allora

$$\int_a^\infty f(x)dx \overset{def.}{=} \lim_{b \uparrow \infty} \int_a^b f(x)dx.$$



Cioè $\int_a^\infty f(t)dt = \lim_{x \uparrow \infty} \underbrace{\int_a^x f(t)dt}_{F(x)}$.

Supponiamo che $f(\cdot) \geq 0$.

f integr. $\overset{def.}{\Leftrightarrow} \int_a^\infty f(x)dx \in \mathbb{R}$.

Criteri di integrabilità (tre):

1) Confronto: $0 \leq f(\cdot) \leq g(\cdot)$ su $[a, \infty[\longrightarrow \begin{cases} g \text{ integrabile} \Rightarrow f \text{ integrabile} \\ f \text{ non integr.} \Rightarrow g \text{ non integr.} \end{cases}$

2) Confronto asintotico: $f(\cdot) > 0$, $g(\cdot) > 0$, $f \sim g$ per $x \to \infty$

$\Rightarrow$ f integrabile se e solo se g integrabile.

["$f \sim g$ per $x \to \infty$" significa $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 1$]

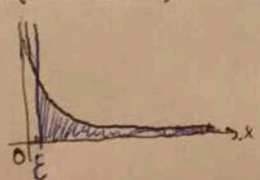
3) Assoluta convergenza: $|f|$ integrabile $\Rightarrow$ f integrabile

$(f \leq |f|)$

$$\left|\int f\,dx\right| \leq \int |f|\,dx < \infty$$

$(\mathcal{E})$ $(\mathcal{E})$

$(|a+b| \leq |a| + |b| \ldots)$

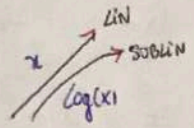


$\left(\int x^{-\alpha}dx = c\,x^{1-\alpha}\right)$

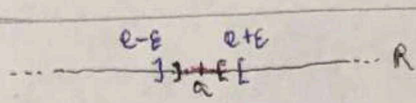
ES. $\forall \varepsilon > 0$, $\int_\varepsilon^\infty \frac{1}{x^\alpha}dx < \infty \Leftrightarrow \alpha > 1$. $\blacksquare$

$\boxed{\text{ANALISI A.}}$ 27/11

$\boxed{\text{EX.}}$ $\sum_{n=2}^{\infty} (-1)^n \underbrace{\log(n)/n}_{\substack{\| \\ a_n \downarrow 0 \\ n\to\infty}} < \infty$ (decrescente)



(LEIBNITZ)



$\boxed{\text{EX.}}$ (TEORICO) $a_n \to a$, $b_n \to b \Rightarrow \underline{a_n b_n \to ab}$.

NOI SAPPIAMO CHE, $\forall \varepsilon/2 > 0$, $\begin{cases} \exists\, n_\varepsilon \in \mathbb{N} \text{ t.c.}, \forall n \geq n_\varepsilon, |a_n - a| < \varepsilon/2 \\ \exists\, m_\varepsilon \in \mathbb{N} \text{ t.c.}, \forall n \geq m_\varepsilon, |b_n - b| < \varepsilon/2 \end{cases}$

$\varepsilon \gtrsim 0$ $(\varepsilon \ll 1)$

$\Rightarrow \forall n \geq \underbrace{\max\{n_\varepsilon, m_\varepsilon\}}_{=: N'_\varepsilon}$, VALGONO ENTRAMBE

FISSIAMO $\varepsilon > 0$. VOGLIAMO TROVARE $N_\varepsilon \in \mathbb{N}$ TALE CHE, $\forall n \geq N_\varepsilon$,

$$|a_n b_n - ab| < \varepsilon .$$

MA, INFATTI: $a_n b_n - ab \equiv (a_n b_n - a_n b) + (a_n b - ab)$

$$\overset{\text{DIS.TR.}}{\Rightarrow} |a_n b_n - ab| \leq |\underbrace{a_n b_n - a_n b}_{a_n(b_n - b)}| + |\underbrace{a_n b - ab}_{b(a_n - a)}|$$

$$= \underbrace{|a_n|}_{\substack{< |a| + \varepsilon \\ \forall n \geq N'_\varepsilon}} \underbrace{|b_n - b|}_{\substack{< \varepsilon/2 \\ \forall n \geq N'_\varepsilon}} + \underbrace{|b|}_{\substack{\text{INDIP. DA} \\ n}} \underbrace{|a_n - a|}_{\substack{< \varepsilon/2 \\ \forall n \geq N'_\varepsilon}}$$

(RICORDIAMO CHE $\underbrace{||a_n| - |a||}_{|a_n| - |a|} \overset{\text{NOTO}}{\leq} |a_n - a| \overset{n \geq N'_\varepsilon}{<} \varepsilon \Rightarrow |a_n| < |a| + \varepsilon$.)

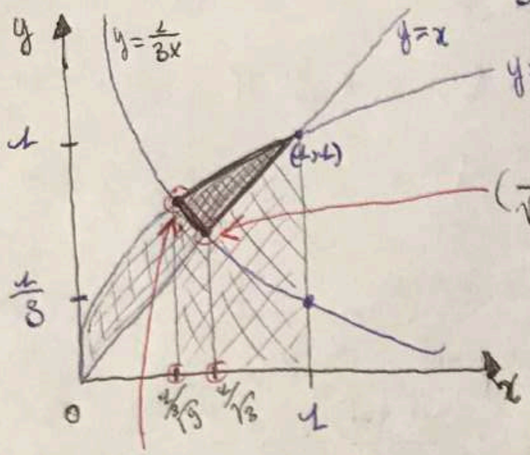
$$< \underbrace{(|a| + \varepsilon)}_{\leq |a| + 1} \varepsilon + |b| \cdot \varepsilon < (\text{"costante"}) \cdot \varepsilon/2 \ \checkmark$$

(es.) 2

DA AGGIUSTARE, MA OK

$\boxed{\text{EX.}}$ CALCOLARE L'AREA (in $\mathbb{R}^2$) COMPRESA TRA (i GRAFICI DI):

$y = x$, $y = \sqrt{x}$ e $y = \dfrac{1}{3x}$, $x > 0$.



$\left(\frac{1}{\sqrt{3}}, \dots\right)$ $\left(\frac{1}{3x} = x\right)$

$\left(\frac{1}{\sqrt[3]{9}}, \dots\right)$ $\left(\frac{1}{3x} = \sqrt{x}\right)$ $\left(\frac{1}{9} = x^3\right)$

$$\int_{1/\sqrt[3]{9}}^{1} \sqrt{x}\,dx - \int_{1/\sqrt[3]{9}}^{1/\sqrt{3}} \left(\frac{1}{3x}\right)dx - \int_{1/\sqrt{3}}^{1} x\,dx = \text{AREA} = \dots$$ ✓ ok

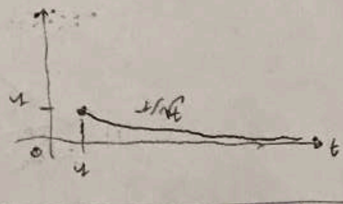
---

$\boxed{\text{"EX."}}$ (SVISTA SU TEMA D'ESAME) $F(x) \doteq \displaystyle\int_{x^2+1}^{x^3+1} f(t)\,dt$, $f: \mathbb{R} \to]0, \infty[$. F è TALE CHE, $\forall x \in \mathbb{R}$,

$F(x) > 0 \iff x^2 < x^3 \iff x > 1$.

$\boxed{\text{EX.}}$ $\displaystyle\int_{-\infty}^{1} \frac{1}{\sqrt{2-x}}\,dx$, cioè $\displaystyle\lim_{a \downarrow -\infty} \int_{a}^{1} \frac{1}{\sqrt{2-x}}\,dx$.

(SARÀ $+\infty$, perché $\displaystyle\int_1^{\infty} \frac{1}{\sqrt{t}}\,dt = \infty \dots$)



$\boxed{\text{EX.}}$ $G(x) \doteq \displaystyle\int_{-1}^{x} \frac{e^t}{t(t+2)^{1/3}}\,dt = \int_{-1}^{x} \frac{e^t}{t\sqrt[3]{t+2}}\,dt$, ~~$x \in \mathbb{R}$~~ ~~$x > -1$~~, $x \in \mathbb{R}$ ~~$x > -2$~~.

$x = 0$ e $x = -2$

PER $t = -2$, SINGOLARITÀ "ANALOGA" A QUELLA CHE HA $\frac{1}{y^\alpha}$ "in $y = 0$" con $\alpha = \frac{1}{3} < 1$ $(]0, 1[)$ $\to$ "IDEA"

UN CONTO SUL FATTO CHE "$G(0) = -\infty$":

$$G(0) \text{ "=" } \int_{-1}^{0} \frac{e^t}{t\sqrt[3]{t+2}}\,dt = \lim_{\varepsilon\downarrow 0}\int_{-1}^{-\varepsilon} \underbrace{\boxed{\frac{e^t}{t\cdot\sqrt[3]{t+2}}}}_{A(t)}\,dt$$

$-1 \qquad -\varepsilon \;\; 0$

$A(t) \sim \underset{\frac{1}{\sqrt[3]{2}}}{\text{cost}}\cdot\frac{1}{t}$, su $]-1,0[$, per $t\to 0^-$

$$\left(\lim_{t\to0^-}\frac{A(t)}{\frac{1}{\sqrt[3]{2}\,t}} = 1\right)$$

[CRIT. DEL CONFRONTO ASINTOTICO "AL FINITO"]

ORA IL PROBLEMA È CHE $\frac{1}{\sqrt[3]{2}}\int_{-1}^{0} \frac{1}{t}\,dt = -\infty$! ✓

(DUNQUE G È DEFINITA SU $]-\infty,0[$)

$\boxed{\text{OSS.}}$ $(a>0,\ a\neq 1)$ $\log_a t \in \mathbb{R}$, $t>0$ "COSÌ COME"

[IL DOMINIO DI $\log_a(\cdot)$ È $]0,\infty[$]

$a^x > 0$, $x\in\mathbb{R}$ ...

$\boxed{\text{EX.}}$ $\sum_{n=1}^{\infty} (-1)^n a_n$ $(\underset{>0}{a_n}\downarrow 0)$ CONV. $\Rightarrow$ $\overbrace{\sum_{n=N}^{\infty}(-1)^n a_n}^{r_N} \to 0$ per $N\to\infty$.

? "DA QUALE N_ε IN POI", $|r_N|<\varepsilon$, DOVE $\varepsilon \doteq 1\% = \frac{1}{100}$?

DIM. CHE

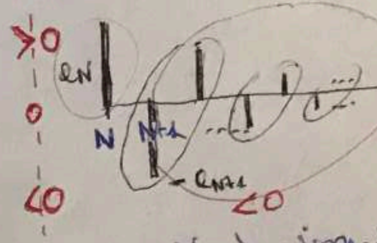
$$r_N = \sum_{n=N}^{\infty}(-1)^n a_n \le a_{N+1}.$$

AD ES, SE N È PARI, $r_N = \underbrace{a_N - a_{N+1} + a_{N+2} - a_{N+3} \cdots}$

$a_N + (-a_{N+1}) + a_{N+2} + (-a_{N+3})$

$< a_N$ per N pari

$< a_{N+1}$

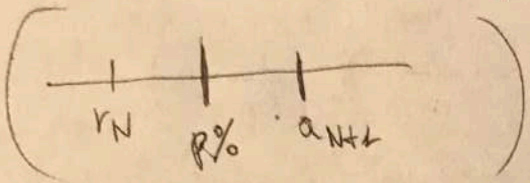


SE N È DISPARI, $r_N = (-a_N) + a_{N+1} + (-a_{N+2}) + a_{N+3} \le r_{N+1} < a_{N+1}$.
$N+1$ PARI

CONCLUSIONE: CHE SIA N PARI O DISPARI (N IGNOTO!),

$$r_N = \sum_{m\geq N} (-1)^m a_m \leq a_{N+1} \; . \quad ✓$$

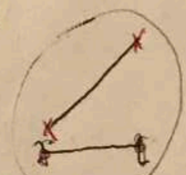
$< p\%$, $p \in]0,1[$ (COND. SUFFICIENTE)



OSS. OTTIMIZZAZIONE: MAX/MIN DI FUNZ. C^1 [INTERVALLI LIMITATI] $]a,b[$, $[a,b[$, $]a,b]$, $[a,b]$

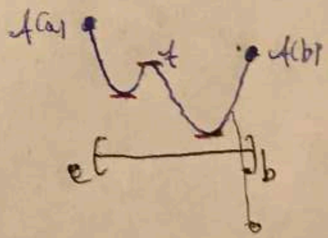
* SU UN INTERVALLO APERTO ($]a,b[$): CALC. DIFFERENZIALE!

(STUDIO DEI PUNTI STAZIONARI)



* SU UN INT. CHIUSO ($[a,b]$): WEIERSTRASS + CALC. DIFF. IN $]a,b[$!

CIOÈ, UNA VOLTA CAPITA L'ESISTENZA,



→ PUNTI STAZ. IN $]a,b[$

→ CONFRONTA COI PUNTI DI BORDO a e b.

* $[a,b[$ o $]a,b]$: CALC. DIFF. IN $]a,b[$ E POI "BORDO" .....

OSS. $(a>1)$ $\quad \dfrac{\sin^2(a^n)}{a^n} \xrightarrow[n\to\infty]{} 0$, $\quad \dfrac{n^\alpha}{a^n} \xrightarrow[n\to\infty]{} 0$ $\quad (\alpha>0)$

(CONFRONTO NUM. E DENOM.)

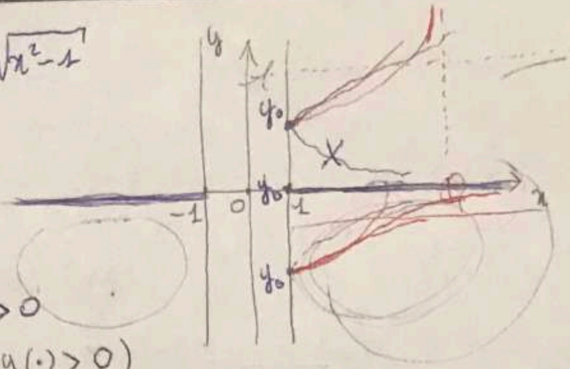
$(0\leq)\sin^2(\dots)\leq 1$

TIPICAMENTE VANNO RAPPORTATI IN MODO "FINE" .....

+ ALTRI COMMENTI FINALI ... ...

ANALISI A - 4/12 $\quad y'(x) = (y(x))^2 \sqrt{x^2-1}$

EX $\begin{cases} y' = y^2 \sqrt{x^2-1} > 0, & |x| > 1 \\ y(\underset{\pm 1}{x}) = y_0 \in \mathbb{R} \setminus \{0\} \end{cases}$



$\lim_{x \to \infty} y(x) = \underset{(\geq 0)}{l} \in \mathbb{R}$, $\underset{(\Rightarrow y(\cdot) > 0)}{y_0 > 0}$ $\rightarrow$ COSÌ COME $y_0 < 0 \Rightarrow y(\cdot) < 0$

EQ. A VAR. SEPARABILI:

$\frac{dy}{dx} = e(x) b(y) \leftrightarrow \frac{dy}{b(y)} = e(x) dx \leftrightarrow \int \frac{1}{b(y)} dy = \int e(x) dx$

PER NOI, $\int \underbrace{\frac{1}{y^2}}_{y^{-2}} dy = \int \sqrt{x^2-1}\, dx \; \dots$

SE SÌ, ALLORA DALL'EQUAZ.: $\lim_{x \to \infty} y'(x) = \infty$ PER $l \neq 0$

IN OGNI CASO, DATO CHE "$y' \propto \sqrt{x^2-1}$", $y(\cdot)$ NON PUÒ AVERE ASINTOTI, OSSIA "ESPLODE". "LO FA A $+\infty$ O PRIMA?"

$\rightarrow$ DIVERSI DA $y \equiv 0$.

$y'(x) = y(x)^2 \sqrt{x^2-1}, \quad x \geq 1$

$\boxed{x \leq -1} \rightarrow -x, \; x \geq 1$:

$y'(-x) = y(-x)^2 \sqrt{x^2-1}$

$\frac{d}{dx} y(-x) = -y'(-x) = -\underbrace{y^2(-x)}_{(y(-x))^2} \sqrt{x^2-1} \dots$

$-y(\cdot)$ CHE EQUAZ. SODDISFA?

$\frac{d}{dx}(-y(x)) = -y'(x) = -y^2 \sqrt{x^2-1} < 0$

(RAGIONAMENTI GENERALI DA RIPRENDERE MEGLIO ... ...)

$\boxed{EX.}$ $\sum_{n=2}^{\infty}(-1)^n\underbrace{\left[\frac{\sqrt{n}+(-1)^n}{n}\right]}_{a_n>0 \quad (n\geq 2)} \longrightarrow \frac{\sqrt{n}+(-1)^n}{n}=\frac{1}{\sqrt{n}}+\frac{(-1)^n}{n}$

$(n=(\sqrt{n})^2)$

$\cancel{\sum_{n\geq 2}^{\infty}(-1)^n\frac{\sqrt{n}}{n}}$ $\cancel{\sum_{n\geq 2}^{\infty}(-1)^n\frac{(-1)^n}{n}}$

$\longrightarrow \cancel{\frac{1}{n}\left[\sqrt{n}+(-1)^n\right]}$

$$=\underbrace{\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}}_{<\infty \ (\text{LEIBNITZ})}+\underbrace{\sum_{n=1}^{\infty}\frac{1}{n}}_{\infty}=\infty \rightarrow \text{DIVERGENTE!}$$

$((-1)^n)^2=1$

$\boxed{EX.}$ $\begin{cases} y''+3y'+2y=2\sin(4x), & x\in\mathbb{R} \\ y(0)=0, \quad y'(0)=0 \end{cases}$ $\left(\begin{matrix}\lambda^2+3\lambda+2=0\\ \lambda_{1/2}=\begin{smallmatrix}-1\\-2\end{smallmatrix}\end{matrix}\right)$

$\begin{cases} y_1(x)=\cancel{c_1}e^{-x} \\ y_2(x)=\cancel{c_2}e^{-2x}\end{cases}$ $\bar{y}(x)=c_1(x)e^{-x}+c_2(x)e^{-2x}$

$$\begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x}\end{vmatrix}=-2e^{-3x}+e^{-3x}=-e^{-3x}$$

$\rightarrow \begin{cases} c_1'(x)=(+e^{3x})\,e^{-2x}\cdot 2\cdot\sin(4x) = 2e^{x}\sin(4x) \\ c_2'(x)=(-e^{3x})\,e^{-x}\cdot 2\cdot\sin(4x) = -2e^{2x}\sin(4x)\end{cases}$ $\leftarrow \left(\frac{1}{-e^{-3x}}=-\frac{1}{e^{-3x}}=-e^{3x}\right)$

$\boxed{\text{DIGRESSIONE.}}$

SIA DATA $\underline{y''(x)+ay'(x)+b=f}$, $\forall a,b\in\mathbb{R}$, $x\in\mathbb{R}$, f CONTINUA, E SIANO (...) $y_1(x), y_2(x)$ DUE SOLUZIONI <u>INDIPENDENTI</u> DELL'<u>OMOGENEA</u> ASSOCIATA

$$z''(x)+az'(x)+b=0.$$

Ciò significa che, $\forall x$, $\underbrace{\begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix}}_{y_1(x)y_2'(x) - y_1'(x)y_2(x)} \neq 0$.

Sappiamo che la generica soluzione dell'equazione è della forma

$$\bar{y}(x) + \text{cost}_1\, y_1(x) + \text{cost}_2\, y_2(x),$$

dove $\bar{y}(x)$ è soluzione particolare dell'equaz. Possiamo trovare $\bar{y}(x)$, ad esempio, col metodo di "variazione delle costanti":

(I) Risolvi / integra

$$\begin{cases} c_1'(x) = \dfrac{-y_2(x)A(x)}{y_1(x)y_2'(x) - y_1'(x)y_2(x)}, & x \in \mathbb{R} \\[2ex] c_2'(x) = \dfrac{y_1(x)A(x)}{y_1(x)y_2'(x) - y_1'(x)y_2(x)}, & x \in \mathbb{R} \end{cases}$$

(II) Poni $\bar{y}(x) = c_1(x)y_1(x) + c_2(x)y_2(x)$.

Quindi, in particolare, $c_1(x) = 2\int \underbrace{e^x}_{\text{integr.}} \underbrace{\sin(4x)}_{\text{deriv.}}\, dx$ (int. parti)...

$$\boxed{y(x) = (c_1(x) + C_1)\, y_1(x) + (c_2(x) + C_2)\, y_2(x)}$$

dai valori iniziali

(ad es. $y(0) = 0$, $y'(0) = 0$)

$\to y'(x) = (c_1'(x) + c_1) y_1'(x) + c_1''(x) y_1(x)$

$+ (c_2(x) + c_2) y_2'(x) + c_2'(x) y_2(x)$

$\rightsquigarrow$ SISTEMA LINEARE 2×2. ("EASY")

OSS. $y'(x) + a(x) y(x) = f(x)$, $y(x_0) = y_0$

PER LE LINEARI, AVETE UN ALGORITMO!

$\rightsquigarrow \boxed{A(x) := \int_{x_0}^{x} a(t)\,dt}$

$\to y_0 \exp(-A(x))$

$\to$ VAR. COSTANTI : $c(x) \exp(-A(x))$

$$y(x) = \exp(-A(x)) \left\{ y_0 + \int_{x_0}^{x} f(t) \exp(A(t))\,dt \right\}.$$

OSS. (SOMIGLIANZA) $y'' + ay' + by = 3e^{5x}$ $2\sin(4x)$, t^3 ...

f DI FORMA "SEMPLICE" (RISPETTO ALLE OPERAZIONI DI DERIVAZIONE)

$y(x) = \alpha e^{\beta x} \to y'(x) = \alpha\beta e^{\beta x} \to y''(x) = \alpha\beta^2 e^{\beta x}$

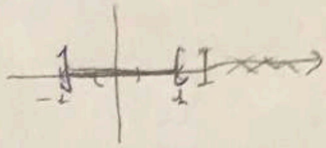
$\cancel{e^{\beta x}[\alpha\beta^2 + a\alpha\beta + b\alpha]}$

$\to \alpha \cancel{e^{\beta x}} [\beta^2 + a\beta + b] = \cancel{e^{5x}}$ $\quad \beta = 5$

$\cancel{\beta^2 + a\beta + b = 1/\alpha^2}$ $\quad \alpha = 1$

$\alpha[25 + 5a + b] = 1$

OSS. (X CASA) 1) $F(x) = \int_0^x \frac{e^{-t^2}}{1-t^2}\,dt$, $x \in \text{Intorno}(0)$.



INT. MASSIMALE DI DEFINIZIONE: $]-1,1[$.

2) DIMOSTRAZIONE CRITERIO DI LEIBNITZ.

3) $\sum_{n=1}^{\infty} \frac{1}{n \cdot \sqrt[n]{n!}} < \infty$. [STIRLING: $\lim_{n\to\infty} \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{n!} = 1$, ...] Li RIVEDIAMO INSIEME

EX DIM. CHE LA FUNZIONE DERIVATA DI UNA FUNZIONE PARI, RISP. DISPARI, È DISPARI, RISP. PARI.

OSS. Senza usare la definizione, ossia il rapporto incrementale, è immediato! Ad esempio, vediamo che $f(x) = f(-x) \Rightarrow -f'(x) = f'(-x)$:

$$f'(-x) = -\frac{d}{dx}\left[f(-x)\right] \equiv -\frac{d}{dx}\left[f(x)\right] = -f'(x).$$

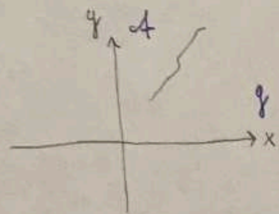
DERIVAZ. FUNZIONE COMPOSTA

✓

EX SIA $f(x) = e^{2x} + x^3$, $x \in \mathbb{R}$.

→ f È INVERTIBILE (SU TUTTO $\mathbb{R}$). SIA $g := f^{-1}$.

→ CALCOLARE $g'(1)$.



[IDEA DELLO SVOLGIMENTO: f È SURIETTIVA PERCHÉ $x \mapsto x^3$ LO È, ED È INIETTIVA IN QUANTO È STRETTAMENTE CRESCENTE: $f'(x) = 2e^{2x} + \underbrace{3x^2}_{\geq 0} > 0$ $(\exp(\dots) > 0)$.

IN PARTICOLARE, OSS. CHE $\boxed{f(0) = 1}$ E CHE 0 È L'UNICO $x \in \mathbb{R}$ T.C. $f(x) = 1$.

ADESSO, INDICATA $g := f^{-1}$, DERIVANDO $(g \circ f)(x) = x$ OTTENIAMO CHE

$g'(f(x)) \cdot f'(x) = 1$, CIOÈ $(f' \neq 0)$ $\boxed{g'(f(x)) = \frac{1}{f'(x)}}$. IN PARTICOLARE,

PER $x = 0$, $g'(1) = \frac{1}{f'(0)} = \frac{1}{2}$. ✓]

ANALISI A. | 11/12/'20

OSS. EQ. DIFF. LINEARI del 2° ORDINE. (COEFF. COSTANTI).

SOLUZIONI INDIPENDENTI DELL'EQ. OMOGENEA ASSOCIATA (↝ WRONSKIANO...):

- $\Delta > 0 \to \lambda_+ \neq \lambda_- \in \mathbb{R} \to e^{\lambda_+ \cdot x}, e^{\lambda_- \cdot x}$
- $\Delta = 0 \to \lambda_+ = \lambda_- = -a/2$ (SE IL POL. È $\lambda^2 + a\lambda + b = 0$) $\to \begin{cases} \exp(-\frac{a}{2}x) \\ x \cdot \exp(-\frac{a}{2}x) \end{cases}$
- $\Delta < 0 \to \begin{cases} \lambda_+ = \alpha + i\beta \\ \lambda_- = \alpha - i\beta \end{cases}$ $(\alpha, \beta \in \mathbb{R}, \beta \neq 0) \to \begin{cases} \exp(\alpha x)\cos(\beta x) \\ \exp(\alpha x)\sin(\beta x) \end{cases}$

OSS. CRIT. LEIBNITZ: $e_n \downarrow 0 \Rightarrow \sum_{n=0}^{\infty} (-1)^n e_n$ CONVERGE (AD UN LIMITE ≥ 0).

"IDEA" DIMOSTRATIVA:

$$\sum_{n=0}^{\infty} (-1)^n e_n = \underbrace{e_0 - e_1}_{b_0 \geq 0} + \underbrace{e_2 - e_3}_{b_1 \geq 0} + \dots\dots + \underbrace{e_{2n} - e_{2n+1}}_{b_n \geq 0} + \dots \quad [e_n \geq e_{n+1}]$$

$$(*) = \sum_{n=0}^{\infty} b_n \quad \text{SERIE A TERMINI POSITIVI (O NULLI)},$$

DOVE APPUNTO $b_n := e_{2n} - e_{2n+1}$. (PER MEGLIO SCRIVERE,

$$b_1 + b_2 + \dots + b_n = e_0 - e_1 + \dots + e_{2n} - e_{2n+1}.)$$

BASTA DIMOSTRARE CHE LA SUCC. DELLE SOMME PARZIALI DI (*) È LIMITATA:

MA, INFATTI,
$$b_1 + b_2 + \dots + b_n = e_0 + \underbrace{(-e_1 + e_2)}_{\leq 0} + \underbrace{(-e_3 + e_4)}_{\leq 0} + \dots + \underbrace{(-e_{2n-1} + e_{2n})}_{\leq 0} \underbrace{- e_{2n+1}}_{\leq 0}$$

$$\leq e_0 \quad \text{(INDIP. DA } n\text{)}. \quad \boxtimes$$

$\boxed{\text{OSS.}}$ "STIRLING" - DIM. CHE $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n!}} < \infty$. IL PROBLEMA È

$\sqrt[n]{n!} = (n!)^{1/n}$ E CHE, "AD ES.", NON VALE $n^n \leq n!$ ....

ANZI, $n! < n^n$!!! . L'IDEA È USARE CHE:

$$\lim_{n\to\infty} \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{n!} = 1 \quad \text{(FORMULA DI STIRLING)}.$$

(def.)
$\Leftrightarrow$ $n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ (PER $n\to\infty$) $\underset{(\dots)}{\Rightarrow}$ $\sqrt[n]{n!} \sim \frac{n}{e}$

$\left(\underset{(\dots)}{\Rightarrow} n! < n^n\right)$ $\Rightarrow$ $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n!}} < \infty$ PERCHÉ

CRIT. CONFRONTO
ASINTOTICO

$$e \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty \;. \checkmark \quad \blacksquare$$

$\boxed{\text{EX.}}$ SIA $f: I \to \mathbb{R}$, $I \subset \mathbb{R}$ INTERVALLO APERTO CENTRATO IN 0 ($I =]-a,a[$, $a>0$) ($\forall x \in I, -x \in I$), f DERIVABILE, E SIA $f': I \to \mathbb{R}$ LA SUA FUNZIONE DERIVATA. DIM. CHE:

$$f \text{ PARI} \Rightarrow f' \text{ DISPARI}.$$

CIOÈ CHE, $\forall x \in I$, SE $f(-x) = f(x)$, ALLORA $f'(-x) = -f'(x)$.

1ª DIM. [DIRETTA]: USANDO DERIVAZ. COMPOSTA PER $x \overset{-\cdot}{\mapsto} -x \overset{f}{\mapsto} f(-x)$,

$$f'(x) = \frac{d}{dx}(\underbrace{f(-x)}_{f(x)}) = f'(-x)\cdot(-1) = -f'(-x) \;. \checkmark$$

[VEDI ANCHE NOTE VOLTA SCORSA]

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2^a DIM.: $\forall x \in I$,

[RAPPORTO INCREMENTALE]

$$\begin{cases} f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} \\ \\ f'(-x) = \lim_{\varepsilon \to 0} \dfrac{f(-x+\varepsilon) - \overset{=f(x)}{f(-x)}}{\varepsilon} \end{cases}$$

"IDEA":

"$-x+\varepsilon = -(x+h)$ CON h E ε DI SEGNI OPPOSTI"

PRENDENDO ε E h TALI CHE $\varepsilon = -h$

RIASSUMENDO,

$$f'(-x) \equiv \lim_{\varepsilon\to 0} \frac{f(-x+\varepsilon) - f(-x)}{\varepsilon}$$

$$= \lim_{-\varepsilon \to 0} \frac{f(-x-\varepsilon) - f(-x)}{-\varepsilon} \qquad \updownarrow = \underbrace{f(-(x+\varepsilon)) - f(-x)}_{\overset{\text{Hyp.}}{=}\ f(x+\varepsilon) - f(x)}$$

$$= -\lim_{-\varepsilon \to 0} \frac{f(x+\varepsilon) - f(x)}{\varepsilon}$$

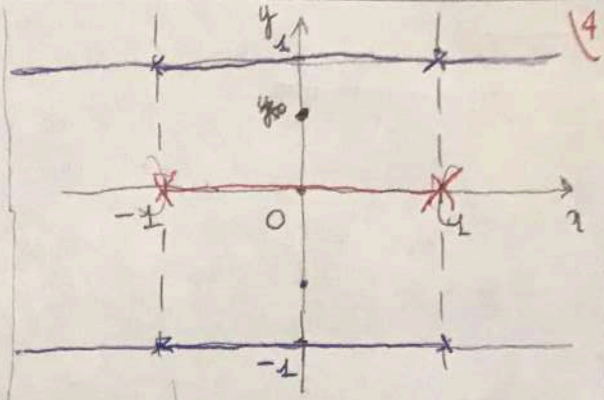
$$= -\lim_{\underset{\varepsilon}{-\varepsilon} \to 0} \frac{f(x+\varepsilon) - f(x)}{\varepsilon}$$

$$= -f'(x).$$

SE $\lim_{-\varepsilon\to 0} \dfrac{f(x+\varepsilon) - f(x)}{\varepsilon} = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ (OK) ... ∎

(EASY)

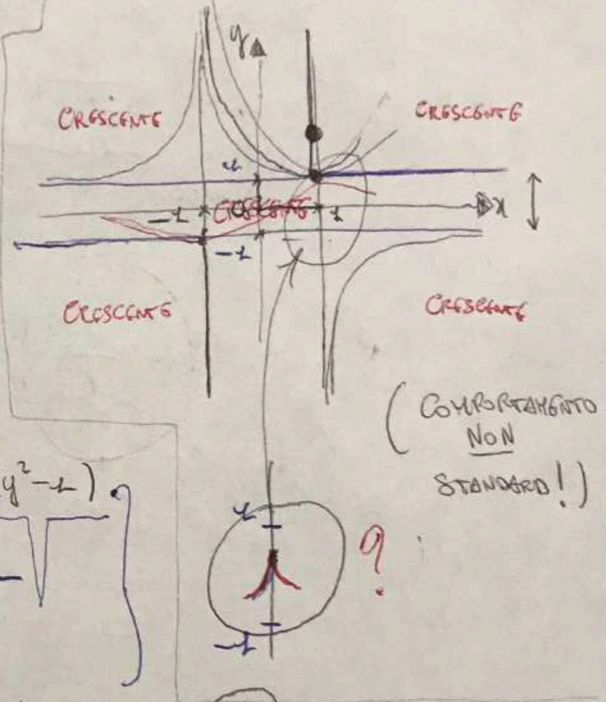
$\boxed{\text{Ex.}}$ $\begin{cases} y' = (y^2-1)/(x^2-1) \,, & x \neq \pm 1 \\ y(x_0) = y_0 & (x_0 \neq \pm 1) \end{cases}$



$y' \geq 0 \iff \begin{cases} y^2-1 \geq 0 \iff y^2 \geq 1 \\ |x| > 1 \end{cases}$ $(y \geq 1 \text{ o } y \leq -1)$

oppure

$\begin{cases} y^2-1 \leq 0 \iff |y| \leq 1 \\ |x| < 1 \end{cases}$



(*) $$y'' = \frac{d}{dx}\left[\frac{y^2-1}{x^2-1}\right]$$

$$= \frac{1}{(x^2-1)^2}\left\{(x^2-1)\frac{d}{dx}(y^2-1) - 2x(y^2-1)\right\}$$

$$\frac{d}{dx}(y^2-1) \equiv \frac{d}{dx}y^2 = 2y\,y'$$

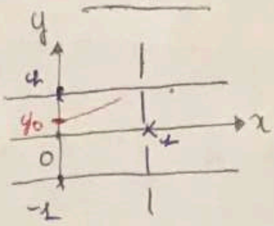
(*) $$\boxed{|y-1|/|y+1| = C\,|x-1|/|x+1|} \,, \quad C \in \mathbb{R}$$

$$\int \frac{dy}{y^2-1} = \int \frac{dx}{x^2-1} \ \ldots$$

$$\left(|x| < 1 \rightarrow \left(\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)\right)' = \frac{1}{1-x^2} \ \ldots \right)$$

$(y(x))$

→ PER $0<x<1$ E, SE $x_0=0$ E $y_0>0$, PER $0<y<1$:



$$\frac{1-y(x)}{1+y(x)} = C\,\frac{1-x}{1+x}\,, \quad C>0$$

$$1-y = C\frac{1-x}{1+x}(1+y) =$$

$$= C\frac{1-x}{1+x} + C\frac{1-x}{1+x}\,y$$

CIOE'

$$1 - C\frac{1-x}{1+x} = \left(1 + C\frac{1-x}{1+x}\right)(1+y)$$

$$\boxed{y(x)} = \frac{1 - C\frac{1-x}{1+x}}{1 + C\frac{1-x}{1+x}} - 1 = \boxed{\frac{-2C\frac{1-x}{1+x}}{1 + C\frac{1-x}{1+x}}}$$

~~Ad es, se y_0 è t.c. $C=1$, $y(x) = \frac{2}{1+\frac{1-x}{1+x}} = 1+x \ldots$~~

$$\frac{\left(1 - C\frac{1-x}{1+x}\right) - \left(1 + C\frac{1-x}{1+x}\right)}{1 + C\frac{1-x}{1+x}}$$

RIVEDERE i CALCOLI

$$\boxed{y(x) = \frac{-2C\frac{1-x}{1+x}}{1 + C\frac{1-x}{1+x}}} \xrightarrow[x\to 1]{} 0 \; ?!$$

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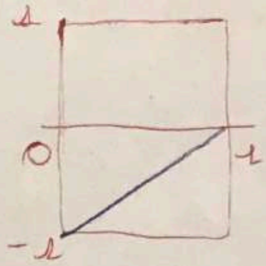
$$1 + C\frac{1-x}{1+x} = \frac{1+x+C(1-x)}{1+x} \quad \dots \quad \Big(\text{SE AD ES. } C = 1,$$

$$y(x) = \frac{-2\,\frac{1-x}{1+x}}{\frac{2}{1+x}} = x-1 \Big)$$



$$\dots \quad \int \frac{1}{y^2-1}\,dy = -\int \frac{1}{1-y^2}\,dy \overset{|y|<1}{=} -\frac{1}{2}\log\left(\frac{1+y}{1-y}\right)$$

$$= \frac{1}{2}\log\left(\frac{1-y}{1+y}\right)$$

$$\longrightarrow \dots \longrightarrow \quad \log\left(\frac{1-y(x)}{1+y(x)}\right) = \log\left(\frac{1-x}{1+x}\right) + 2K$$

$\left(|x|<1,\ |y|<1\right)$

$$\longrightarrow \quad \frac{1-y(x)}{1+y(x)} = C\,\frac{1-x}{1+x} \qquad \text{ok}$$

$\boxed{\text{EX.}}$ $f(x) = e^{2x} + x^3$, $x \in \mathbb{R}$. (QUINDI $f: \mathbb{R}_x \to \mathbb{R}_t$)

DIM. CHE f È INVERTIBILE SU TUTTO $\mathbb{R}$ (INIETTIVA E SURIETTIVA)

E, POSTO $g := f^{-1}$, CALCOLARE $g'(1)$.

(STIAMO PARLANDO DI QUELL'UNICA FUNZ. $g: \mathbb{R}_t \to \mathbb{R}_x$ TALE CHE

$\forall x \in \mathbb{R},\ g(f(x)) = x$ E, $\forall t \in \mathbb{R},\ f(g(t)) = t$.)

Giovanni Alberti (Prof. ordinario)

Univ. di Pisa

!per esercizi validissimi!

Analisi 1 / Analisi A / Appunti

Digressione "a caso"

EpocCam Viewer ..

App per usare smartphone come web cam.

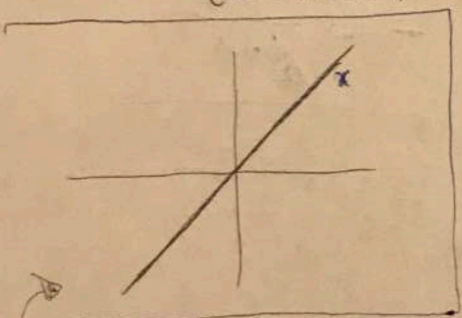
Iniettività: $\forall x,\ f'(x) = \underbrace{2e^{2x}}_{>0} + \underbrace{3x^2}_{\geq 0} > 0$. Strett. cresc.

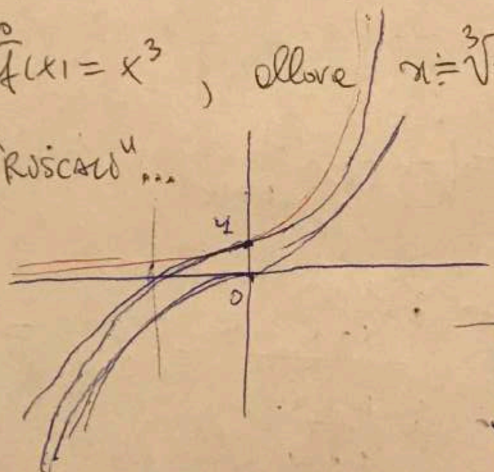
Suriettività: $x \mapsto x^3$ lo è ....

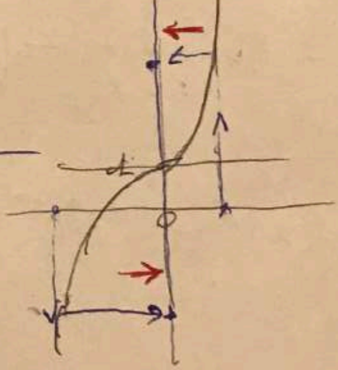
$\forall t \in \mathbb{R},\ \exists x \in \mathbb{R}$ t.c. $f(x) \equiv e^{2x} + x^3 = t$?

Sì: se fosse stato $f(x) = x^3$, allora $x \doteq \sqrt[3]{t}$ è t.c. $f(x) = t$.

Altrimenti, (nel nostro caso) "ruscali"...







$$(g \circ f)(x) = x \to 1 = \frac{d}{dx}(g \circ f)(x)$$

$$= g'(f(x)) \cdot f'(x)$$

$\to \forall x \in \mathbb{R},\ \boxed{g'(\overset{t}{f(x)}) = \frac{1}{\underset{g(t)}{f'(x)}}}$. Ora, chi è $x \in \mathbb{R}$ tale che $f(x) = 1$?

$\boxed{x = 0}$: segue $g'(1) = \frac{1}{f'(0)} = \frac{1}{2}$. ✓

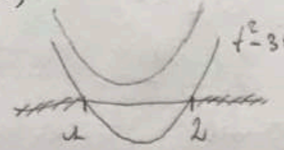
$(f(0) = 1)$

[vedi comunque note volta scorsa]

ANALISI A $(18/12/'20)$ [ULTIMA]

EX. $F(x) \doteq \int_1^x \frac{1}{\sqrt{t^2-3t+2}}\,dt$, $x \in \cancel{\mathbb{R}}$ (?) . $x \leq 1$. $(F(1)=0)$ $(]-\infty,1])$

$\forall t \in \mathbb{R},\ t^2-3t+2 = (t-1)(t-2) \to F(x) = \int_1^x \frac{1}{\sqrt{(t-1)(t-2)}}\,dt$.

$\to$ 

$t<1 \ \vee\ t>2 \iff t^2-3t+2 > 0$.

$\forall x<1,\quad F(x) = -\int_x^1 \frac{1}{\sqrt{t^2-3t+2}}\,dt \in \mathbb{R}_-$ $(\grave{E} < 0)$ $\left(\int_0^\varepsilon \frac{1}{\sqrt{y}}\,dy \in \mathbb{R}\ \ \forall \varepsilon>0 \dots \right)$.

(OSS $F(x) \xrightarrow[x\to-\infty]{} -\infty$). ~~X~~CRESCENTE, ECC. ...

OSS. FORMULA/SVILUPPO DI TAYLOR, DI ORDINE $n \in \overset{\shortparallel}{\mathbb{N}}$ $\{0,1,2,\dots\}$ CON RESTO/ERRORE DI LAGRANGE O (TAYLOR) PEANO, DELLA FUNZIONE $f \in C^{n+1}(]a,b[;\mathbb{R})$, PER $x \to x_0 \in]a,b[$:

$$f(x) \underset{x\to x_0}{=} \underbrace{\sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k}_{\text{(POLINOMIO)}} + \underbrace{\frac{f^{(n+1)}(\xi_{x_0})}{(n+1)!}(x-x_0)^{n+1}}_{o_{x\to x_0}(|x-x_0|^n)}, \quad \xi_{x_0} \text{ "TRA" } x \text{ e } x_0$$

INDETERMINATO!

$(f^{(0)} \equiv f\ ,\ 0! := 1)$

"INDETERMINATO" $\to$ $o_{x\to x_0}(|x-x_0|^n)$

$(\xi_{x_0} \xrightarrow[x\to x_0]{} x_0)$

LA DIFFERENZA STA NEL FATTO CHE PER IL RESTO DI PEANO BASTA $f \in C^n$

ES. $n=1 \to$ $f \in C^1 \to$ PER $x \to x_0$ (IN $]a,b[$),

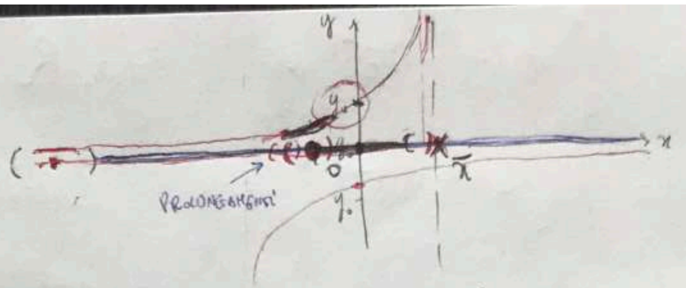
$$f(x) = f(x_0) + f'(x_0)(x-x_0) + o_{x\to x_0}(|x-x_0|)$$

(INFATTI, $\frac{f(x)-f(x_0)}{x-x_0} \xrightarrow[x\to x_0]{} f'(x_0)$)

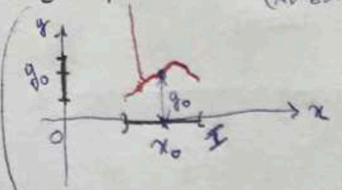
$f \in C^2 \to f(x) = f(x_0) + f'(x_0)(x-x_0) + \overbrace{\frac{f''(\xi_{x_0})}{2}(x-x_0)^2}^{o_{x\to x_0}((x-x_0)^2)}$.

LA COMPONENTE "IMPORTANTE" È IL POLINOMIO DI TAYLOR, NON TANTO IL RESTO, L'ERRORE, CHE DOVRÀ INFATTI ESSER "PICCOLO".

Ex. $\begin{cases} y' = y^2 \log(y^2+1) \\ y(0) = y_0 \in \mathbb{R} \end{cases}$



$(*)\ \begin{cases} \boxed{y'(x) = a(x)\, b(y(x))} \\ y(x_0) = y_0 > 0 \ \text{(AD ES.)} \end{cases}$



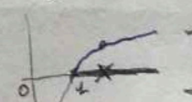
$a(\cdot)$ CONTINUA IN UN INTORNO (APERTO) DI x_0, SIA I E $b(\cdot)$ È C^1 IN UN INTORNO (APERTO) DI y_0

$\Rightarrow$ IL PROBLEMA DI CAUCHY (*) AMMETTE UNA ED UNA SOLA SOLUZIONE $y(\cdot)$ DEFINITA SU UN INTORNO DI x_0 CONTENUTO IN I.

CONDIZIONI DEL TEOREMA: (PIÙ CHE) OK! QUINDI $\exists !\, y(\cdot)$ ALMENO IN UN INTORNO DI 0. PIÙ PRECISAMENTE, PER $y_0 = 0$, $y(\cdot) \equiv 0$ È SOLUZ. SU TUTTO $\mathbb{R}$.

DATO CHE LE CONDIZIONI DEL TEOREMA VARREBBERO PER OGNI ALTRO $x_0 \neq 0$, NESSUNA SOLUZIONE $y(\cdot)$ DI $y'(x) = y(x)^2 \log(y(x)^2+1)$ POTRÀ "TOCCARE" $y(\cdot) = 0$, OSSIA SARÀ > 0 O < 0.

COMUNQUE, $\forall x$ "DOV'È DEFINITA", $y'(x) > 0$ ($y(x) \neq 0$!), DUNQUE $y(\cdot)$ È STRETT. CRESCENTE. $\left[\log(1 + \underset{>0}{\dots}) > 0 \right.$:



$\left.\right]$

(*) DIMOSTRIAMO ORA CHE, $\forall y_0 > 0$ (FISSATO), LA CORRISP. SOLUZIONE $y(\cdot)$ AMMETTE COME INTERVALLO MASSIMALE DI DEFINIZIONE UNA SEMIRETTA DEL TIPO $]-\infty, \bar{x}[$, $\bar{x} > 0$.

$\rightarrow$ SE PER ASSURDO $y(x) \underset{x \downarrow -\infty}{\longrightarrow} l > 0$ (l DEVE ESISTERE!), ALLORA DALL'EQUAZ.

$$y'(x) = y(x)^2 \cdot \log(y(x)^2+1) \underset{x \downarrow -\infty}{\longrightarrow} l^2 \cdot \log(l^2+1) > 0,$$

(PER CONTINUITÀ)

CHE È ASSURDO PERCHÉ $y \equiv l$ ASINTOTO ORIZZONTALE PER $x \downarrow -\infty$ $\Rightarrow y'(x) \underset{x \downarrow -\infty}{\longrightarrow} 0$.

$\Rightarrow$ $y(x)$ ESISTE $\forall x < 0$ ED HA ASINT. ORIZZ. $y \equiv 0$.

→ PER x "LONTANO" DA 0, $x>0$? DALL'EQUAZIONE (ricordando $y(\cdot)\neq 0$),

$$\frac{y'(x)}{y(x)^2\log(y(x)^2+1)} = 1 \quad (*) \quad \Longrightarrow \int_{y_0}^{y(x)} \frac{dy}{y^2\log(y^2+1)} = x \left(= \int_0^x 1\,dt\right), \text{ O MEGLIO}$$

$$\left[(*) \int_0^x 1\,dt = \int_{y(0)}^{y(x)} \frac{dy}{(\cdots)} \right]$$

$$\boxed{x = \int_{y_0}^{y(x)} \frac{dt}{t^2\log(t^2+1)}} \quad (y_0>0)$$

, ADESSO DEDUCIAMO CHE $y(\cdot)$ NON PUÒ ESSER DEFINITA SU TUTTO $]0,\infty[$, PERCHÉ ALTRIMENTI, $\left(\text{dunque } \lim_{x\uparrow\infty} y(x) = \infty\right)$

PER $x\uparrow\infty$, $\Longrightarrow$ "$\int_{y_0}^{\infty} \frac{dt}{t^2\log(t^2+1)} = \infty$", CHE NON È VERO PERCHÉ

$$\int_{y_0}^{\infty} \frac{1}{t^2\log(t^2+1)}\,dt < \infty, \quad \checkmark$$

$$\left[\int_{y_0}^{\infty} \frac{1}{t^2\log(t^2+1)}\,dt = \cancel{\int_0^1 (\cdots)\,dt} + \int_{y_0}^{\infty} (\cdots)\,dt \right. \quad \leq \frac{1}{\log(2)}\int_1^{\infty} \frac{1}{t^2}\,dt < \infty$$

$$\frac{1}{t^2\log(t^2+1)} \leq \frac{1}{t^2\log(2)} \iff \log(t^2+1) \geq C = \log(2)$$

$$\left(\log(1+t^2) \underset{t\downarrow 0}{=} t^2 + o(t^2) \right.$$
SAREBBE ANDATA PEGGIO ...)

$\boxed{\text{OSS.}}$ PER $y' = y\log(y^2+1)$ NON SAREBBE STATA LA STESSA COSA...

$$\left(\frac{1}{t\log(t^2+1)} \text{ VS } \frac{1}{t^\alpha}, \quad \alpha>1 \quad (\alpha\downarrow 1) \quad : \right.$$

$$\Longrightarrow \frac{\frac{1}{t\log(t^2+1)}}{\frac{1}{t^\alpha}} \equiv \frac{t^\alpha}{t\log(t^2+1)} = \frac{t^{\overset{>0}{(\alpha-1)}}}{\log(t^2+1)} \underset{t\uparrow\infty}{\uparrow\infty}$$

CIOÈ $\frac{1}{t\log(t^2+1)}$ TENDE A ZERO PIÙ LENTAMENTE DI OGNI $\frac{1}{t^\alpha}$, $\alpha>1$)

.... QUINDI?

(4

$\boxed{\text{EX.}}$ $F(x) = \int_0^x \log(1+|t|)\, e^{-t^2}\, dt \ , \quad x \in \ldots \mathbb{R} \ldots$

$f(t) := \log(1+|t|)\, e^{-t^2}, \ t \in \mathbb{R}, \ \begin{cases} f(t) = 0 \Leftrightarrow t = 0 \\ f(t) > 0 \ \forall t \neq 0 \end{cases}$

$\left(\exp(\ldots) > 0, \ \log(1+\alpha) > 0 \ \forall \alpha > 0 \right)$

$F(0) = 0$, $F(x) > 0 \ \forall x > 0$, F dispari, ecc.. (FACILE IN QUESTO CASO)

PER L'ESAME:

→ VA SAPUTO "IL GIUSTO DI TUTTO", E QUINDI AVERE (E SAPER FAR VALERE) UNA "BUONA" VISIONE D'INSIEME ED UNA "BUONA" CAPACITÀ DI RAGIONAMENTO ("ORDINATO", COME MINIMO).

→ PER LA PARTE "PRATICA", PROBABILI STUDI QUALITATIVI DI FUNZIONI INTEGRALI E/O SOLUZIONI DI EQUAZIONI DIFFERENZIALI (a variabili separabili).

→ BASTERÀ TELECAMERA E FOGLIO DI CARTA CON PENNA.

→ ISCRIVETEVI!

→ STATE TRANQUILLI, NON È ~~DA~~ RICHIESTA ANCHE UNA PERFORMANCE DI VELOCITÀ.

... AUGURI PER TUTTO!
